GraN-GAN: Piecewise Gradient Normalization for Generative Adversarial Networks 🚽 🛧 Samsung Al Center-Toronto Vin Bhaskara^{*1}, Tristan A. A.^{*1,2,3}, Allan Jepson¹, Alex Levinshtein¹

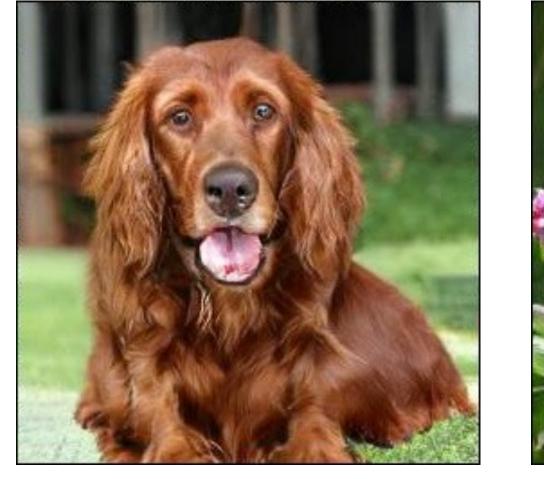


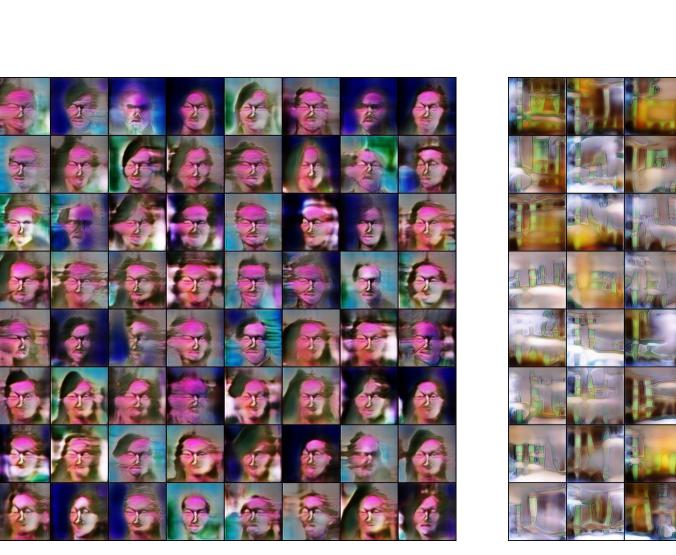
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Introduction

- Generative Adversarial Networks (GANs) can be quite effective in unsupervised image generation.
- > Despite their effectiveness, GANs are hard to train (e.g., mode collapse, training divergence).

(Images taken from StyleGAN and BigGAN)





 \triangleright Recall: we wish to learn generator G, such that x = G(z) looks like a real data sample $x \sim P_{data}$

E.g., with the classical minimax objective (Jensen-Shannon divergence):

 $\min_{C} \max_{D} E_{x \sim P_{data}}[\log D(x)] + E_{z \sim P_{z}}[\log(1 - D(G(z)))]$

The generated images and the training dynamics of the generator network G(z) are dependent on the gradients of the loss function L_G :

$$\nabla_{x}L_{G}(D(x)) = \nabla_{D}L_{G}(D(x)) \nabla_{x}D(x)$$

Loss function gradient: defined by the GAN objective (choice of distributional divergence)

Examples:

- Cross-entropy (CE) loss
- Non-saturating (NS) CE loss
- Wasserstein/Hinge losses

Constraints to stabilize the gradients sent to G in training, ensure D does not get too far ahead of G by regularization, enforce theoretical properties (e.g., WGAN)

"Input-gradient" of Discriminator: function of the network architecture and constraints

Examples:

- gradients $\nabla_x D(x)$:
- Gradient Penalty
- Spectral Norm
- GraN (Ours)

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• Architectures (ResNet, DCGAN) Constraints on the Lipschitz constant and/or the input

Our focus in this work

Previous works

- Gradient Penalties (GPs):
- Soft-constrain the norm of the input-gradient of discriminator/critic
- penalty imposes a two-sided constraint on the grad norm:

$P_1(x) = (||\nabla_x D$

Zero-centered gradient penalty:

 $R_1(x) = ||\nabla_x D(x)||_2^2$

- Spectral Normalization (SN):
- Per-layer 1-Lipschitz constraint on the discriminator/critic using an estimate of the largest singular value $\sigma(W_i)$ of weight matrix W_i :

 $W_i \leftarrow W_i / \sigma(W_i)$

- Drawbacks:
- GPs do not guarantee exact enforcement and their domain must shift to catch-up to G in training.
- SN enforces layer-wise 1-Lipschitzness but can cause gradient

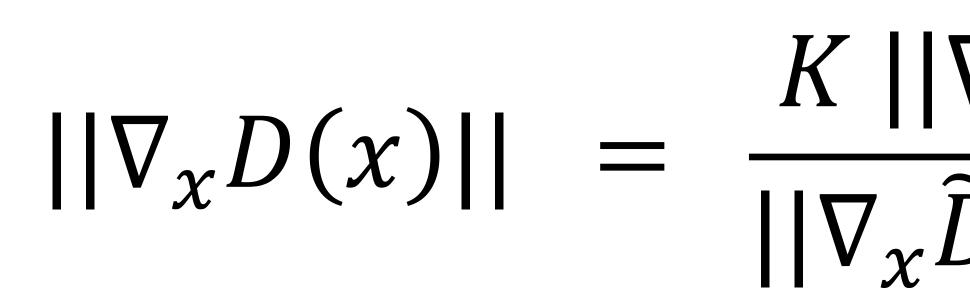
Our method: GraN or Gradient Normalization

When the discriminator/critic is a ReLU network, we can guarantee bounded gradients and piecewise *K*-Lipschitzness by defining the normalized discriminator/critic D(x) as:

> **Discriminator output** before normalization

$$D(x) = \widetilde{D}(x) \frac{k}{\|x\|}$$

 \succ This guarantees a local K-Lipschitz constraint and bounds the gradient norm almost everywhere in x since



Wasserstein-GANs enforce a Lipschitz Constant (LC) = 1. The P_1

$$|x||_2 - 1)^2$$

attenuation due to progressively shrinking (best) LC with depth.

Normalizer \approx

 $K || \nabla_x \widetilde{D}(x) ||$ $V_x \widetilde{D}(x) ||^2 + \epsilon$

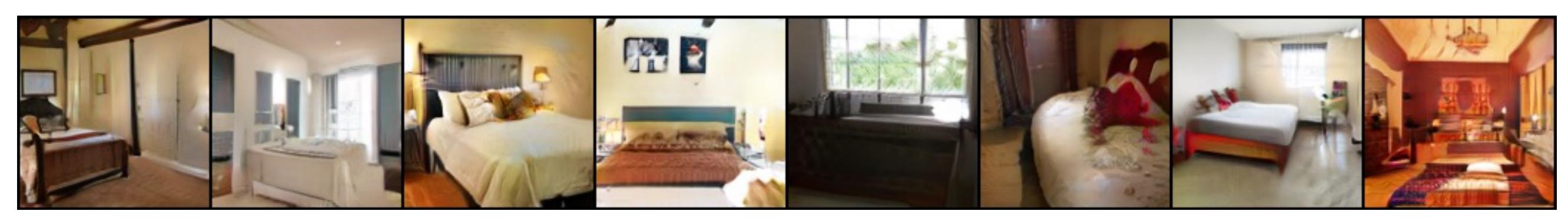
$$\nabla_x \widetilde{D}(x) ||^2 < K$$

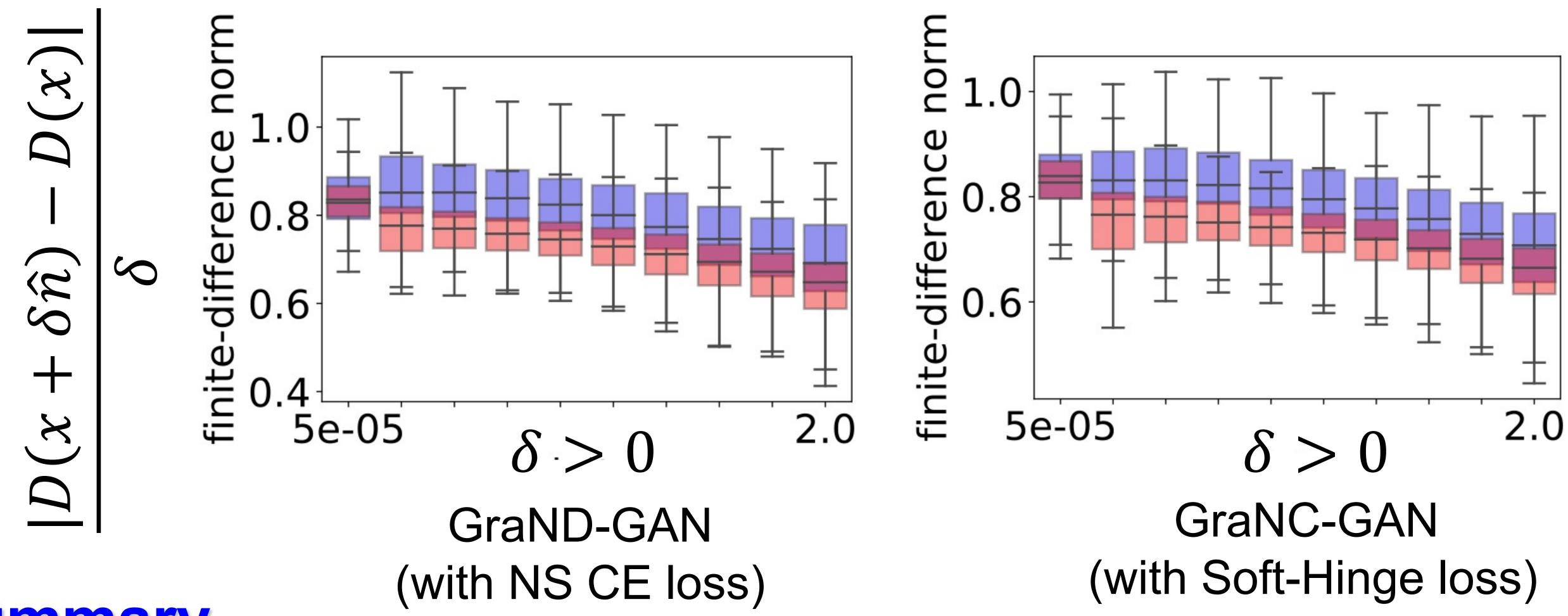
$$\widetilde{D}(x) ||^2 + \epsilon$$

Results on Unconditional Image Generation



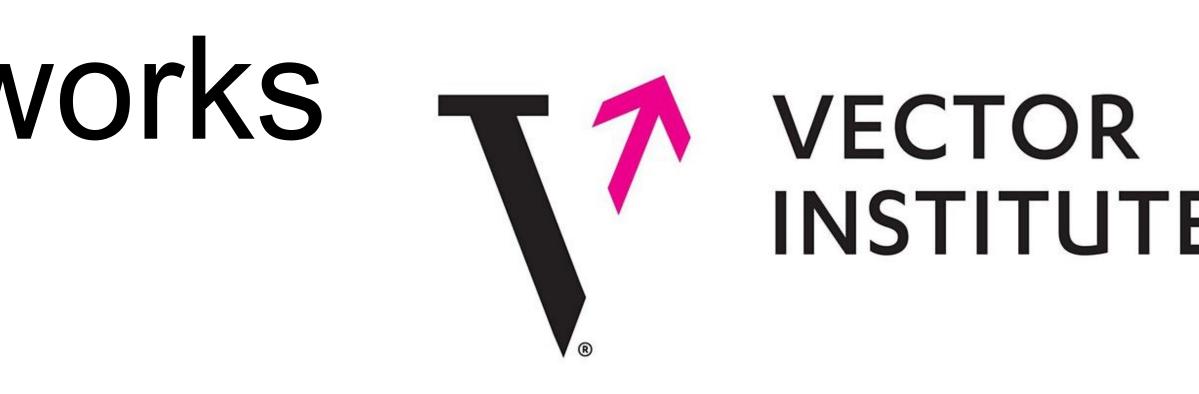






Summary

datasets and loss types.





 \succ We evaluate image generation across multiple datasets (CIFAR-10, CIFAR-100, STL-10, Celeb-A, LSUN Bedrooms) and GAN losses. Check our paper for more detailed results.

WGAN-GP (FID: 13.6)

SNGAN (FID: 13.2)

GraND-GAN (FID: 10.8)

 \triangleright Despite our method enforcing local K-Lipschitzness in theory, empirically the finite-difference grad norms are well-behaved even for large steps δ along $\hat{n} = \nabla_x D(x)$ on CIFAR-10 with K = 0.83:

> We introduced GraN for piecewise linear discriminators/critics, which ensures bounded input-gradients and guarantees a tight local K-Lipschitz constraint almost everywhere, yet does not constrain individual layers. GraN results in improved GAN performance across